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Defining the We-map Reference Frames and Providing Mathematical Expressions for Transformation Relations

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ABSTRACT

We-map is an interactive mobile map that can be easily communicated and applied on personal electronic devices, such as personal computers and mobile phones. Therefore, the study of direction systems and coordinate systems is critical, and exploring reference frames is essential in direction and coordinate systems. Despite its significance, existing research on We-map lacks specific solutions for the exploration of reference frames is indispensable for the establishment of accurate direction and coordinate systems. In this paper, we endeavor to address this gap by elucidating the significance of We-map reference frames, defining them with mathematical constraints, summarizing their nature and characteristics, deriving their transformation relationships and representing them through mathematical formulars and equations. Our work contributes to the fundamental theory of We-map and provides valuable systems and support for the mathematical foundation of We-map, map production, and platform development. Ultimately, this research serves to advance the development of We-map.

Key words: We-map; reference frames; spatial direction relation; coordinate system

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1 Introduction

We-map[1], an interactive mobile map that facilitates user interaction and communication on personal mobile devices, has enabled map enthusiasts and users to actively participate in the creation and publication of map content. Current research on We-map has primarily concentrated on such as dissemination and communication[2-3], symbols[4-5] and design[6-8], We-map in the perspective of the post-modernist philosophy[9-10], as well as We-map orientation method[11], with inadequate attention given to the exploration of We-map reference system.

Reference frames are of utmost importance in the field of cartography and Geographic Information Science (GIS), as they not only offer precise positioning assistance but also enable users to navigate

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with a profound sense of direction[12-16]. Moreover, these frames serve as the foundational basis for describing spatial direction relations, playing a crucial role in supporting this aspect of the field[16-19]. Their close connection with coordinate system construction fosters mutual dependence and leads to synergistic advancements[20]. It is evident that reference frames hold a pivotal position in both cartography and GIS.

In the specific context of We-map, the investigation of the reference frames not only complements the foundational theory but also provides essential support for We-map production and facilitates advancements in We-map platforms. Consequently, the exploration and research of reference frames in We-map are of paramount significance and warrant thorough attention.

Despite the explicit definitions and descriptions for Absolute Reference Frames (ARF) and Relative Reference Frames (RRF), the definition, mathematical constraints, and properties of We-map Reference Frames (WMRF) currently remain ambiguous, demanding urgent exploration and clarification. The existing reference frames can be broadly categorized into two types: ARF and RRF. The former, whether presented as coordinate systems[21-23] or within the context of spatial direction relations[19], possess well-defined and strictly mathematically constrained characteristics. Conversely, relative reference frames exhibit clear definitions and descriptions within the context of the spatial direction relations[16], with some studies even introducing mathematical constraints to enhance comprehension[17-18].The We-map Reference Frames (WMRF) consist of ARF and RRF, but this paper focuses on the RRF.

Within the scope of We-map, a matter of utmost significance pertains to the transformation relationship between ARF and RRF, primarily due to the imperative need for unambiguous, absolute, and precise. Nevertheless, the current absence of well-defined definitions and mathematical constraints for RRF has given rise to ambiguities and uncertainties in spatial position expressions when employing relative reference frames. To ensure the attainment of accurate and unambiguous spatial position descriptions, it is imperative to establish the transformation relationship between ARF and RRF and articulate it using mathematical language.

To recapitulate, the central focus of the current study revolves around the definition of RRF with the incorporation of mathematical constraints and the subsequent derivation of the transformation relationship between ARF and RRF, expressed through mathematical formulations. This crucial matter demands immediate attention and resolution within the context of the research.

2 Definition and Description for RRF

Reference frames play a foundation role in analyzing the motion of object or targets, while coordinate systems provide a means to describe the spatial positioning of features with respect to the reference frame's origin[20]. For instance, the barycentric reference frame of the solar system is employed to study planetary motion, while the Earth-centered reference frames are used to examine terrestrial and Earth-space movements. In the specific context of We-map, the We-map reference frame is selected to explore the movement of We-map users in geographic space, providing essential support for We-map production. Moreover, the Cartesian coordinate system O-XYZ is utilized to express a point's position relative to the origin point O in three-dimensional coordinates (X, Y, Z). Although reference frames and coordinate systems exhibit different expression, they share fundamental similarities, and thus, this paper dose not distinguish between the two.

The WMRF, consisting of both ARF and RRF, establishes the fundamental spatial framework governing users' movements within geographic space and serve as the backbone for We-map production, as previously discussed. The ARF is associated with spatial objects or features situated on Earth's surface and remains fixed relative to Earth (or Earth's

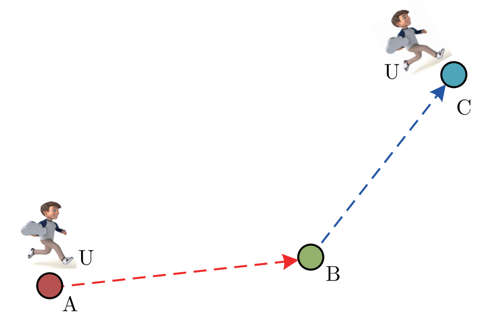

Figure 1 Schematic of different reference objects with the same target

surface). In contrast, the RRF encompasses spatial objects or features on Earth's surface, whose positions vary in relation to We-map users, comprising both Self-Centered Reference Frame (SCRF) and Fixed Reference Frame (FRF).

In a particular scenario, the positions of spatial objects exhibit change as perceived by users at distinct temporal instances. As illustrated in Fig. 1, when user U moves from point A through B to ultimately reach point C, the user undergoes motion relative to Earth's surface, while the positions of targets A, B and C remain constant relative to the Earth's surface. However, from the vantage point of user U, the position of target B experiences variation. Initially situated ahead of U at point A, target B transitions to a position behind U upon reaching point B.

In the context of We-map, various reference frames are utilized, including ARF like Earth-centered reference frames and geodetic coordinates systems, which offer practicality in certain scenarios. For instance, professional cartographers or relevant departments may utilize global reference frames in the production of standard maps. Extensive research has been conducted on global reference frames, with specific parameters and values detailed in reference[24]. However, this paper will not extensively delve into global reference frames; Rather, it will concentrate on the exploration of RRF, which primarily comprise SCRF and FRF.

2.1 Self-Centered Reference Frame (SCRF)

The self-centered reference frame can be succinctly defined based on the following criteria.

1) Origin: It denotes the position of a rigid body at a specific moment, subject to alterations due to the movement of the rigid body, thereby representing relative motion.

2) Scale: The scale of the reference frame is a composite concept, encompassing two fundamental aspects: extension and intension, which involve spatial, temporal, and semantic components. Its intension includes three key elements: breadth, granularity, and frequency, collectively forming a 3×3 matrix[25]. In the context of this paper, the scale of the reference frame primarily pertains to the spatial and temporal aspects within the scale extension. Specifically, the scale represents a temporal or length characteristic of a process, observation, or model. Consequently, within the SCRS, the scale is defined as meters (SI units) that align with the time coordinates of TCG (Geocentric Coordinate Time) in the local Earth-centered frame.

3) Orientation: In the geodetic coordinate reference system, the initial orientation is anchored to the 1 984.0 orientation of the Bureau International de l'Heure (BIH), and its evolution over time is governed by the Earth's horizontal tectonic motion without net rotation[21, 23-24]. However, within the SCRS, the concept of orientation diverges from that of the geodetic reference coordinate system. Here, orientation pertains to the user's spatial perception and their delineation of directions within their individual spatial context. Specifically, orientation in the SCRS involves partitioning directions based on support of the SCRS. This entails establishing a spatial direction model that employs the origin of the SCRS to record or describe spatial direction relations. For example, in Fig. 2, the X-axis represents the forward direction, the Y-axis denotes the rightward direction, while the Z-axis indicates the upward direction.

4) Temporal Evolution: In the geodetic coordinate reference system, temporal evolution primar-

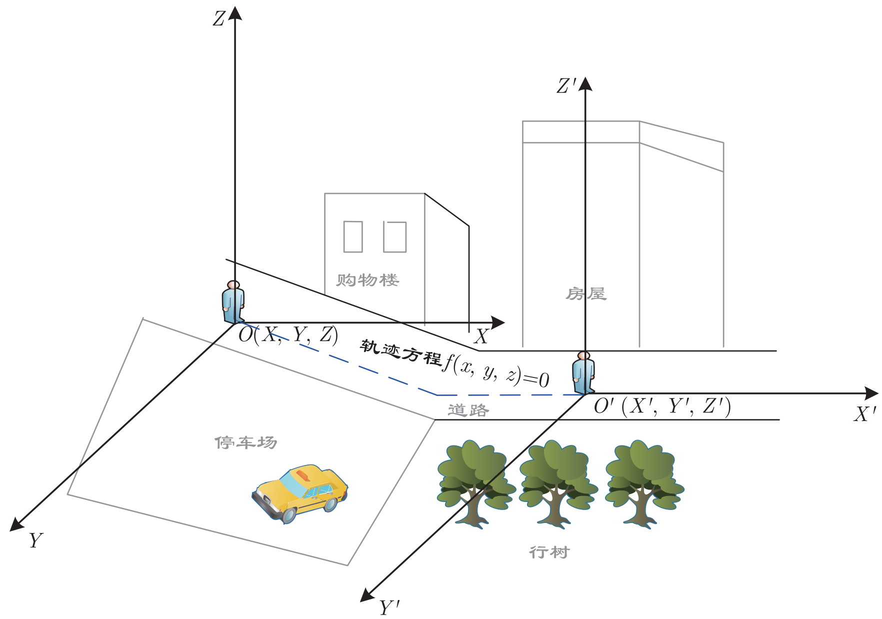

Figure 2 Self-centered reference frame

ily concerns the assurance of orientation changes over time. In contrast, within the context of this paper, temporal evolution refers to the movement or displacement of the reference frame's origin over time. This movement is governed or constrained by the trajectory equation (or motion equation) of the origin point.

The provided definitions are applicable to a Cartesian coordinate system, which comprises its origin and axis, as depicted in Fig. 2.

1) Origin: The position O of the rigid body at a specific moment.

2) X-axis: Coincides or aligns with the direction of the rigid body's forward movement through the point O.

3) Y-axis: Perpendicular to the X-axis and points towards the right side of the rigid body.

4) Z-axis: Perpendicular to both the X-axis and Y-axis, creating a left-handed coordinate system by being orthogonal to the OXY plane.

2.2 Fixed Reference Frame (FRF)

In the context of the FRF, the term "fixed" denotes the relative stability of the features and targets incorporated within the reference system. For instance, an immobile house located in front of a road remains fixed, irrespective of external alterations. Nevertheless, it is important to note that the FRF belongs to the category of the Relative Reference Frame (RRF). In this context, the term "relative" refers to the non-fixed nature of the origin within the reference system. The precise definition is presented as follows:

1) Origin: Refers to the spatial feature nearest to the user within the SCRS at a specific moment in time.

2) Scale: The scale within the SCRS adopts meters length unit (SI units), aligned with the TCG of the local Earth-centered frame.

3) Orientation: Determined by the user's selected spatial feature as a reference target (i.e., the origin), establishing the directional relations between other spatial features and the origin within the SCRF.

4) Temporal evolution: Primarily focuses on the dynamic movement of the origin over time, involving the computation the spatial feature closest to the origin. This temporal evolution is governed or restricted by the trajectory equation (or motion

equation) of the origin and the utilization of the shortest distance calculation model.

The definitions provided above pertain to a Cartesian coordinate system, wherein the origin and axes are precisely determined as illustrated in Fig. 3.

1) Origin: Refers to the position O of the spatial feature nearest to the rigid body at a specific moment.

2) Z-axis: Aligns with the normal vector of the reference ellipsoid passes through the point O.

3) X-axis: Oriented perpendicular to the Z-axis, and its direction is towards the minor axis of the reference ellipsoid.

4) Y-axis: Perpendicular to both the Z-axis and X-axis, forming a left-handed coordinate system as it remains orthogonal to the OXZ plane.

In this section, it is important to note that the reference ellipsoid employed aligns with the one utilized in the China Geodetic Coordinate System (CGCS), and its constant definitions can be found in Literature [22].

3 Properties, Features, Similarities and Differences for We-map

3.1 Properties

1) Determinism: The determinism of the SCRF is evidenced by the rigid body's motion equation $f = f(t)$, expressed in three-dimensional space as $f(t) = x(t)\ i + y(t)\ j + z(t)\ k$. If $f = f(t)$ is known, a corresponding trajectory equation for the rigid body, denoted as $f\ (x, y, z) = 0$, can be established with an expression such as $z = f(x, y)$. This implies that any point on the trajectory equation $f\ (x, y, z) = 0$ can be selected as the reference origin to construct the SCRF, thus exemplifying the determinism of the SCRF. Furthermore, considering the nearest spatial target to the user, represented as B_n, and the calculation model for the user's position relative to the nearest spatial target, denoted as M_n, it is observed that under equivalent conditions, when $f = f(t)$ is known and the user's position's trajectory equation is $f\ (x, y, z) = 0$, the relationship $M_n\{f(x, y, z) = 0\} = B_n$ is guaranteed, further showcasing the determinism of the FRF.

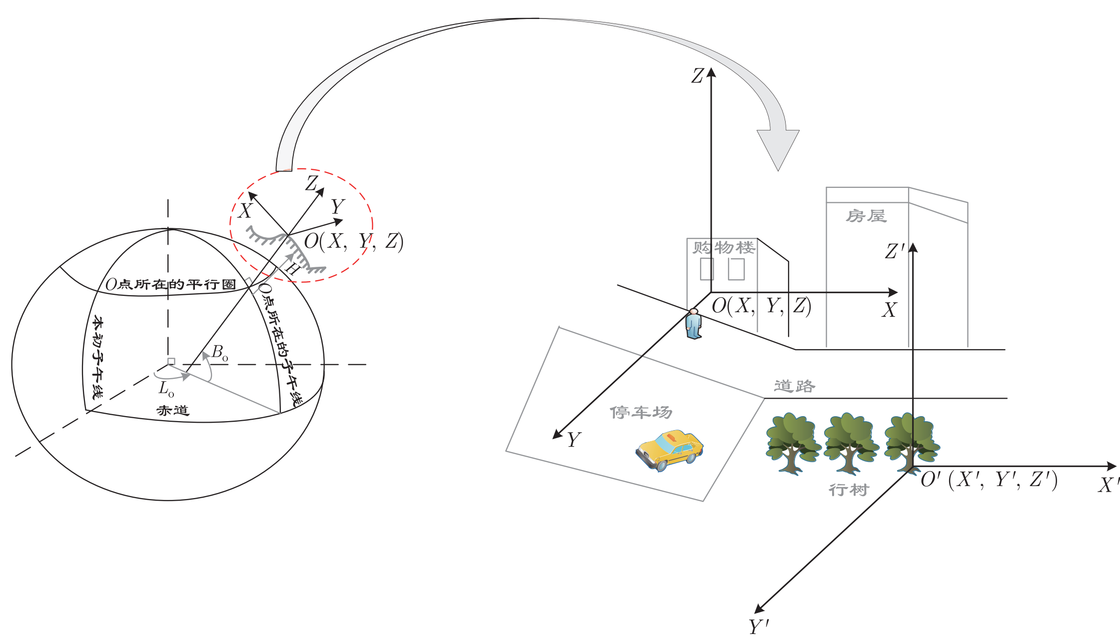

Figure 3 Fixed reference frame

2) Transitivity: Demonstrating the transitivity property of the SCRF, consider the set of user's main directions denoted as $D = \{D_1,\ D_2,\ D_3,\ D_4,\ \cdots,\ D_n, n = 2^{k+1}, k \geq 1\}$, and let G_i represent a spatial target. The calculation function for spatial directions is represented as f. Under the condition that $f(G_1,\ G_2,\ S_F)\ D_i$ and $f(G_2,\ G_3,\ S_F)\ D_i$ for the spatial direction triplets, it logically follows that $f(G_1,\ G_3,\ S_F)\ D_i$ is also satisfied. This property clearly exemplifies the transitivity of the SCRF.

3) Relativity: The concept of relativity in SCRF encompasses two fundamental aspects. Firstly, the origin of the SCRF experiences relative motion and is not fixed at a specific location. Hence, as the position of the origin changes, the SCRF undergoes corresponding transformations. Secondly, the trajectory equation exhibits relativity with respect to the motion equation is also affected and undergoes corresponding alterations accordingly.

4) Fixed nature: FRF exhibits an inherent property known as "fixed nature", which is characterized by the consistent and unchanging relative spatial relationships between various spatial features. As depicted in Fig. 3, irrespective of whether the shopping mall is chosen as the reference origin, the spatial relationship between the road and the shopping mall remains constant. This exemplifies the inherent stability and fixed nature of the FRF, where the spatial configurations maintain their relative positions over time and do not undergo significant alterations.

3.2 Features

1) Diversity: WMRF displays a diverse range of characteristics, underscoring its multifaceted nature. Firstly, WMRF is not confined a singular ARF but rather encompasses both ARF and RRF. Secondly, WMRF not only serves as a reference system for traditional coordinate systems but also serves as a fundamental reference frame for calculating and resolving spatial directional relationships. The versatility of WMRF extends beyond conventional mapping applications and accommodates virtual maps, including gaming maps, which may encompass spatial feature beyond Earth's surface. By embracing this diversity, WMRF demonstrates its adaptability and applicability across a broad spectrum of geospatial scenarios.

2) Complexity: The establishment and utilization of the WMRF entail intricate and multifaceted processes that defy simplicity. The intricacy arises from a convergence of diverse factors, including the rigid body trajectory equations, motion equations, and the geodetic reference ellipsoid. The interplay of these various contributes to the complexity inherent in the WMRF's definition, computation, and solution procedures. Consequently, a comprehensive understanding of the WMRF necessitates a nuanced examination of its constituent components and the intricate relations between them. Addressing the intricacies involved WMRF construction remains an essential aspect of its exploration and analysis.

3) Flexibility: The WMRF demonstrate notable flexibility, characterized by dynamically determination relative to ARF. This dynamism is driven by the real-time motion state of the rigid body, necessitating adaptable definition of the reference system's origin and constraints to cater to the evolving demands of users. Moreover, the use of ARF may encounter limitations, particularly in area with restricted GPS or BeiDou satellite positioning coverage, leading to challenges in accurately identifying specific floors or penetrating buildings. In such scenarios, the selection and construction of the reference system demand a high degree of adaptability and flexibility. The ability of the WMRF to accommodate various constraints and dynamic conditions empowers its applicability in diverse real-world situations, bolstering its significance in the context of We-map applications.

3.3 Similarities and differences

Regardless of whether it is the SCRF, or the FRF, both are aligned with common objective, with the WMRF striving to address the unique requirements of the We-map application. These reference systems play a role in We-map cartography and

Table 1 Similarities and differences between We-map reference systems

Reference frame	Target	Object of reference	Scenarios	Target users	Ease of use
ARF	We-map	Earth's surface	Relevant cartographic departments produce standard maps	PMCs	√
				NPMCs	×
				NPMCs	×
SCRF	We-map	Users themselves	Used to record events that occurred to oneself	PMCs	√
				NPMCs	√
				NPMCs	√
FRF	We-map	Fixed objects	Create a We-map to provide pathfinding experience for other users	PMCs	√
				NPMCs	√
				NPMCs	√

Note: ARF refers to the absolute reference frame, PMCs represent professional map producers, and NPMCs represent non-professional map producers. The checkmark "√" indicates ease of use, while the symbol "×" indicates a certain level of difficulty in usage.

the development of associated software platforms. However, it is important to recognize that while pursuing this common goal, notable difference exist in their reference frame designs, scale ranges, application scenarios, and target users. As the target users of the WMRF encompass diverse entities, it becomes imperative to thoroughly examine and classify their usability based on the specific needs of individual user groups. An in-depth analysis of these distinctions will facilitate the optimal tailoring of the WMRF to different users, enhancing its efficacy and applicability in diverse real-world setting.

Tab. 1 presents a comprehensive summary of the similarities and distinctions among different WMRF. As discussed in Section 1, the SCRF revolves around the user as it center, while the FRF centers on a spatial feature that remains fixed and closed to the user. In both SCRF and FRF, the reference origins are established relative to the user. Despite the reference origin in FRF being associated with a fixed spatial feature, it's constraints are contingent upon the user's movement, thereby classifying both SCRF and FRF as relative reference systems.

On the other hand, substantial distinctions emerge in the application scenarios of these reference systems. The ARF embraces a comprehensive approach, featuring heighted precision, thus gaining favor among Professional Map Producers (PMCs) seeking meticulous cartographic outputs. In contrast, the SCRF and FRF target Non-Professional Map Producers (NPMCs). The SCRF caters to users who generating We-map to record personal events, characterized by less stringent precision requirements, such as plotting travel routes or marking points of interest during trips. Conversely, the FRF is tailored to provide pathfinding experiences for other users, demanding a higher level of precision than SCRF but lower than ARF. The FRF still demands specific spatial feature coordinates to facilitate user navigation effectively. These divergent application scenarios signify the adaptability and versatility of WMRF, enabling their optimization to suit the distinct needs of diverse user group.

The usability of various WMRF exhibits differences depending on the target users. Particularly in the context of standard map production, the ARF is well-suited for Professional Map Producers (PMCs), but may present challenges for Non-Professional Map Producers (NPMCs). Hence, the ARF primarily caters to PMC users, making map production relatively easier for this group. When it comes to recording personal events, both PMCs and

NPMCs find using the SCRF more accessible. In the case of generating We-map to provide navigation experiences for other users, NPMCs may encounter certain difficulties using the FRF, but compared to using the ARF, the difficulty is significantly reduced. Consequently, SCRF and FRF encompass a broader range of map production users, not limited solely to PMCs, but also accommodating NPMCs, while the ARF primarily focuses on serving PMC users and may, to some extent, overlook the needs of NPMCs. These distinctions in usability emphasize the importance of considering diverse user groups when selecting the appropriate frame for We-map production.

4 Transformation Relations between Reference Frames

In the context of We-map production, whether based on the SCRF, the FRF, or with the support of the ARF, it is crucial to consider the transformation relationship between these reference frames. The WMRF can adopt different reference systems based on diverse scenarios, while ensuing that the spatial objects or features depicted remain fixed. Moreover, the spatial objects at a specific location will not undergo changes relative to the Earth's surface due to user movements; rather, they only change with respect to the user's perspective. Therefore, it is necessary to determine the transformation relationship between these reference frames to serve various user groups and accommodate different application scenarios effectively. By addressing these transformation relationships, We-map can provide accurate and reliable spatial information, catering to the needs of its users and enhancing the overall user experience.

To facilitate the computation of the transformation relationship between SCRF, FRF and ARF, it is imperative to establish clear definitions of relevant concepts and symbols. Let vector $\boldsymbol{p}$ represents a specific spatial object. Within the context of the We-map's particular reference system, a set of orthogonal basis ($\boldsymbol{x}_\mathbf{c}$, $\boldsymbol{y}_\mathbf{c}$, $\boldsymbol{z}_\mathbf{c}$) exists. As a result, vector $\boldsymbol{p}$ possesses coordinates in this basis as outlined below

$$\boldsymbol{p} = [\boldsymbol{x}_\mathbf{c}, \boldsymbol{y}_c, \boldsymbol{z}_c] \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \boldsymbol{x}_\mathbf{c} p_1 + \boldsymbol{y}_c p_2 + \boldsymbol{z}_c p_3 \quad (1)$$

Where, the expression $[p_1, p_2, p_3]^\mathrm{T}$ corresponds to the coordinates of vector $\boldsymbol{p}$ in the given basis. The specific values of these coordinates are contingent upon both the vector $\boldsymbol{p}$ itself and the selection of the basis.

4.1 Transformation relations between SCRF and the absolute reference frame

In this section, we seek the transformation relationship between vector $\boldsymbol{p}$, representing a spatial target from the perspective of a rigid body, and its coordinates in the SCRF denoted as $\boldsymbol{p}_\mathbf{c}$, and the ARF denoted as $\boldsymbol{p}_\mathbf{w}$. The ARF is denoted as O_w—$X_w Y_w Z_w$, while each user is represented as $U = \{i \in N | U_1, U_2, \ldots, U_i\}$, and their corresponding SCRF is denoted as U_i—$X_i Y_i Z_i$.

As shown in Fig. 4, there exists only one the ARF, which remains constant irrespective of the number of users. However, numerous SCRF can be attributed to the U_i users. Each SCRF generates different coordinate values based on the respective user. Ensuring unambiguous, absolute, and accu-

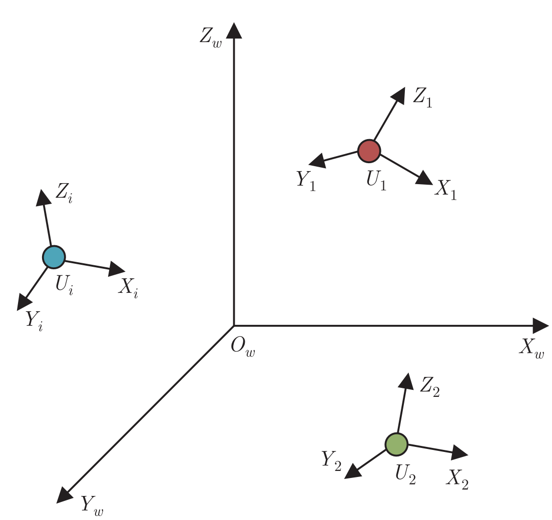

Figure 4 Relations between SCRF and the absolute reference frame

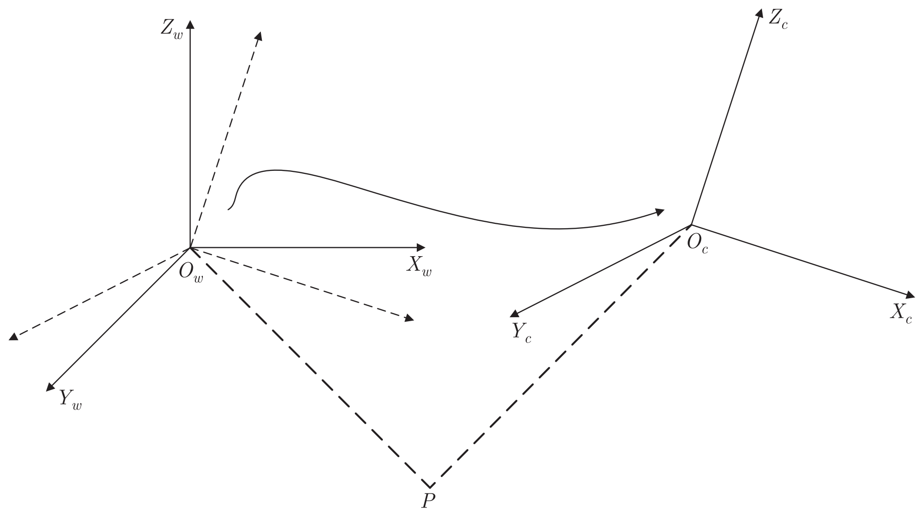

Figure 5 Transformation relation between the SCRF and the absolute reference frame

rate position data is vital. Therefore, it is imperative to calculate the transformation relationship between SCRF and the ARF to guarantee the unambiguous and accurate representation of position data.

The transformation between the two reference frames involves a sequence of operations: rotation and translation, as illustrated in Fig. 5. Throughout this transformation process, the length and angles of vector $\boldsymbol{p}$ remain unaltered, irrespective of whether it is expressed in the SCRF or the ARF. Therefore, it is essential to consider both rotation and translation as two fundamental motion states in this context.

Furthermore, even though there are multiple distinct SCRFs denoted as U_i for different users, it is a adequate to derive the transformation relationship between one SCRF and the ARF. This is attributed to the fact that the transformation relationship among various SCRFs and the ARF is consistently characterized by rotations and translations. Therefore, in this section, we opt to consider one SCRF as a representative example and proceed to calculate the transformation relationship between this specific SCRF and the ARF.

1) Rotation: Let us suppose an orthonormal basis $[\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3]$ that undergoes a rotation to become $[\boldsymbol{e}_1', \boldsymbol{e}_2', \boldsymbol{e}_3']$. For a given vector $\boldsymbol{p}$ (where the vector itself remains unchanged despite the rotation of the reference frame), its coordinates in the two reference frames are $[p_1, p_2, p_3]^{\mathrm{T}}$ and $[p_1', p_2', p_3']^{\mathrm{T}}$, respectively. As the vector itself remains constant, in accordance with the coordinates definition, the equation is expressed as follows

$$[\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3]\begin{bmatrix}p_1\\p_2\\p_3\end{bmatrix} = [\boldsymbol{e}_1', \boldsymbol{e}_2', \boldsymbol{e}_3']\begin{bmatrix}p_1'\\p_2'\\p_3'\end{bmatrix} \tag{2}$$

By left-multiplying both sides of equation (2) with the matrix $\boldsymbol{R}^{-1}$ (the inverse of the rotation matrix $\boldsymbol{R}$), we obtain

$$\begin{bmatrix}p_1\\p_2\\p_3\end{bmatrix} = \begin{bmatrix}\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_1' & \boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_2' & \boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{e}_3'\\ \boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_1' & \boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2' & \boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_3'\\ \boldsymbol{e}_3^{\mathrm{T}}\boldsymbol{e}_1' & \boldsymbol{e}_3^{\mathrm{T}}\boldsymbol{e}_2' & \boldsymbol{e}_3^{\mathrm{T}}\boldsymbol{e}_3'\end{bmatrix}\begin{bmatrix}p_1'\\p_2'\\p_3'\end{bmatrix} \stackrel{\text{def}}{=\!=} \boldsymbol{R}\boldsymbol{p}' \tag{3}$$

Where $\boldsymbol{R}$ is the rotation matrix, a special orthogonal matrix, and its inverse (which is equivalent to the transpose) represents the opposite rotation. Therefore, we have

$$\boldsymbol{p}' = \boldsymbol{R}^{-1}\boldsymbol{p} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{p} \tag{4}$$

Clearly, $\boldsymbol{R}^{\mathrm{T}}$ depicts the opposite rotation.

2) Translations: In addition to rotation, translation serves as another form of transformation between two reference frames. Considering a vector $\boldsymbol{p}$

in the SCRF, after undergoing a rotation (described by $\boldsymbol{R}$) and a subsequent translation $\boldsymbol{t}$, it transforms into $\boldsymbol{p}'$. By combining rotation and translation together, we arrive at

$$\boldsymbol{p}' = \boldsymbol{R}^{-1}\boldsymbol{p} + \boldsymbol{t} = \boldsymbol{R}^{\mathrm{T}}\boldsymbol{p} + \boldsymbol{t} \tag{5}$$

Where, $\boldsymbol{t}$ represents the motion vector.

In this section, we denote the coordinates of vector $\boldsymbol{p}$ in the SCRF and the ARF as $\boldsymbol{p}_{\mathbf{c}}$ and $\boldsymbol{p}_{\mathbf{w}}$, respectively. The transformation relationship between $\boldsymbol{p}_{\mathbf{c}}$ and $\boldsymbol{p}_{\mathbf{w}}$ can be expressed as follows

$$\boldsymbol{p}_w = \boldsymbol{R}_c^w \boldsymbol{p}_c + \boldsymbol{t}_c^w \tag{6}$$

Where $\boldsymbol{R}_c^{\mathrm{w}}$ denotes the transformation from coordinates in the SCRF to the ARF; and $\boldsymbol{t}_c^w$ represents the vector pointing from the origin of the absolute reference frame to the origin of the SCRF, and it is selected in the ARF.

Similarly, if we denote the transformation from the coordinates in the ARF to the SCRF as $\boldsymbol{p}_{\mathbf{c}}$ and $\boldsymbol{p}_{\mathbf{w}}$, the relationship can be expressed as follows

$$\boldsymbol{p}_c = \boldsymbol{R}_w^c \boldsymbol{p}_w + \boldsymbol{t}_w^c \tag{7}$$

Where $\boldsymbol{R}_c^w = (\boldsymbol{R}_w^c)^{-1} = (\boldsymbol{R}_w^c)^{\mathrm{T}}$.

In summary, Eq.(5) presents the transformation relationship between the SCRF and the ARF, facilitating conversions between the two reference systems through rotations and translations. Eq.(6) and Eq.(7) provide precise expressions for the transformation from the SCRF to the ARF and from the ARF to the SCRF, respectively.

4.2 Transformation relation between FRS and ARF

As discussed in Section 2.2, it becomes apparent that the FRF also exhibits a connection to the reference ellipsoid. Therefore, the transformation relationship between FRF and ARF encompasses not only the conversion between spatial rectangular coordinate systems but also incorporates the transition between spherical coordinates and spatial rectangular coordinates. This entails addressing two key aspects: ① The transformation relationship between spatial rectangular coordinate systems within different reference systems; ② And the conversion relationship between the spherical coordinate system and the spatial rectangular coordinate system.

To establish the transformation relationship between the FRF and the ARF, a comprehensive examination of various coordinate systems is essential. First, the transformation of spatial rectangular coordinate system U_i (X, Y, Z) within the FRF to the spatial rectangular coordinate system $O(X, Y, Z)$ in the ARF must be carefully considered. Subsequently, the transformation between $O(X, Y, Z)$ and the spherical coordinate system S (r, θ, φ) requires through analysis.

In the AFR, the local coordinate system is defined based on the reference ellipsoid that best fits the local geoid in a least-squares sense, exemplified by the 1980 Xi'an coordinate system, which relies on the geodetic origin. On the other hand, the local coordinate system in the FRF pertains to the space object nearest to the user, serving as the coordinate origin. Notably, this coordinate origin may not necessarily coincide with the geodetic origin. Therefore, the selection of a reference ellipsoid that precisely fits the local geoid in a least-squares sense is not obligatory in the RFR. Instead, it suffices to focus only on the transformation relationship between U_i (X, Y, Z) and the geocentric coordinate system $O(X, Y, Z)$.

While both the ARF and the FRF belong to local coordinate systems transitioning to the geocentric coordinate system, they exhibit significant distinctions. The exploration and clarification of these distinctions are fundamental to understanding the transformation process and are paramount in effectively implementing the FRF to ARF conversion in We-map and related applications.

In the context of the FRF, this part aims to elucidate the transformation process from the spatial rectangular coordinate system U_i (X, Y, Z) to the spatial rectangular coordinate system O (X, Y, Z) within the ARF. The transformation relationship between O (X, Y, Z) and the spherical coordinate system S (r, θ, φ) is thoroughly examined. Fig. 6 provides a visual representation of the transforma-

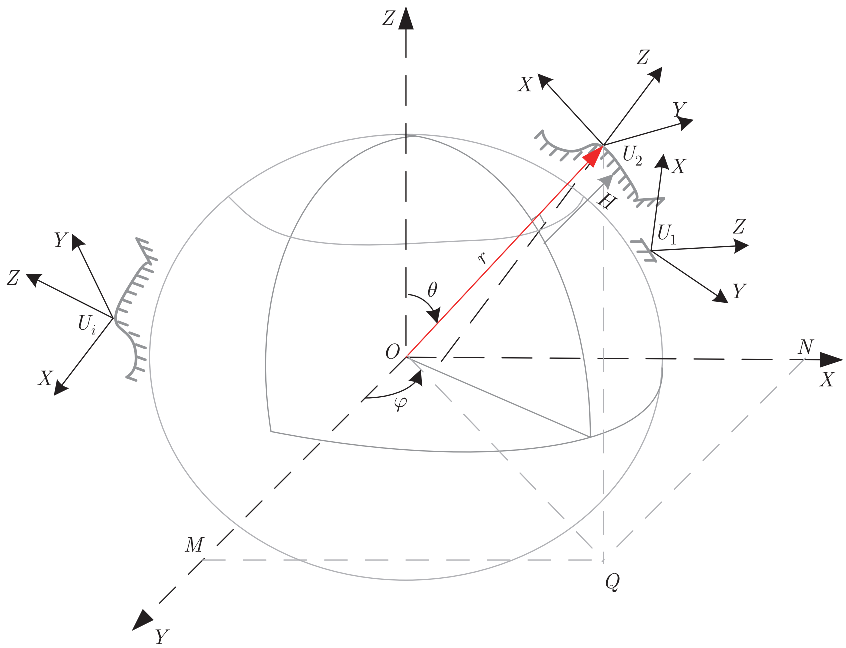

Figure 6 Transformation relation between the FRF and the absolute reference frame

tion process, highlighting its distinctions from the previously discussed Section 4.1.

The key distinction between the transformation relationship of the SCRF and the ARF, as explored in Section 4.1, and that of the FRF and ARF lies in their respective constraints. While the SCRF to ARF transformation is subject to the constraints imposed by the shortest distance calculation model. Once the position of the nearest spatial object to the user is computed and the coordinate system U_i (X, Y,Z) is established, its transformation relationship with ARF aligns with the description in Section 4.1.

Therefore, this section primarily focuses on expressing the transformation relationship between the spatial rectangular coordinate system O (X, Y,Z) and the spherical coordinate system S (r, θ, φ). The insights derived from this examination will contribute to the comprehensive understanding of the FRF to ARF conversion, further advancing the field of We-map and related applications.

Let us consider point P as a spatial object, characterized by its position represented in two distinct coordinate systems: the conventional Cartesian coordinate values (x, y,z) within the $O(X, Y,Z)$ reference frame, and the spherical coordinates (r, θ, φ). The latter is denoted by three ordered real numbers, with each parameter serving a specific purpose. Firstly, r denotes the distance from point P to the origin of the $O(X, Y,Z))$ coordinate system, effectively representing the radius; Secondly, θ represents the angle formed between the line segment OP and the positive Z-axis; Lastly, φ corresponds to the angle between the projection of the line segment OP on the O—X—Y plane and the positive Y-axis.

In this section, our primary focus centers on elucidating the transformation relationship between spatial Cartesian coordinates (x, y,z) and spherical coordinates (r, θ, φ). The transformation process involves two key aspects: the transformation from spherical coordinates to spatial Cartesian coordinates, and the transformation from spatial Cartesian coordinates to spherical coordinates. Established and well-validated formulars for these transformations are readily accessible in existing literature, as demonstrated in Eq.(8) and Eq.(9). These formulars have been extensively studied and proven to provide accurate and reliable results, rendering them suitable for adoption in the context of our investigation.

$$\begin{cases} x = r \cdot \sin\theta \cdot \sin\varphi \\ y = r \cdot \sin\theta \cdot \cos\varphi \\ z = r \cdot \cos\theta \end{cases} \tag{8}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arccos(z/r) \\ \varphi = \arctan(x/y) \end{cases} \tag{9}$$

Where, the transformation relationship between the FRF and the ARF is comprehensively described by Eqs.(5), (8), and (9). Eq.(5) facilitates the seamless transformation of coordinates between different spatial Cartesian coordinate system to another. On the other hand, Eqs.(8) and (9) govern the transformations from spherical coordinates to spatial Cartesian coordinates, and from spatial Cartesian coordinates to spherical coordinates, respectively. The combined utilization of these equations ensures a holistic and accurate mapping between the FRF and ARF, accounting for the conversions between various reference systems. Each equation plays a pivotal role in establishing the connection between the two frames, ensuring the smooth integration of their spatial representations.

5 Conclusion

The investigation of We-map reference frames bears considerable significance in enhancing the fundamental theories of We-map, We-map cartography, and the development of We-map platforms. In this paper, we have provided a comprehensive definition of We-map reference frames, encompassing essential mathematical constraints. We have meticulously outlined the distinctive properties and characteristics of We-map reference frames, deriving into the transformation relationship between absolute and relative reference frames. Our primary focus remains directed towards:

1) The rationality of We-map reference frames has been demonstrated, thereby advancing the theoretical research of We-map and filling the gap in We-map theory exploration. Thus, this scholarly endeavor contributes valuable references for the broader domain of the We-map studies.

2) Within the study, we introduce and define We-map reference frames consist of three types: SCRF and FRF. We focus on the establishment of the relative reference frames (SCRF and FRF) for We-map and subjected them to rigorous mathematical constraints.

3) A comprehensive summary of the properties and characteristics of SCRF and FRF is presented, encompassing four distinct properties and three key characteristics. Notably, FRF demonstrates an additional property compared to SCRF, termed, “Flexibility”, which adds to the versatility of the reference system.

4) Crucially, we derive and express the transformation relationships between diverse reference frames using mathematical language. This essential foundation provides invaluable support for We-map cartography and software development.

The next step of research will build upon the foundation laid in this paper to explore the specific implementation of We-map reference frames. With the validation of the definitions and derived mapping relationships presented in this study, corresponding We-map cartography algorithms will be designed. Subsequently, concrete experimental data will be analyzed to determine whether they can provide tangible instances of support for We-map cartography.

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