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Recent Advances in the Geodesy Data Processing

Jianjun ZHU[1], Leyang WANG[2], Jun HU[1], Bofeng LI[3], Haiqiang FU[1], Yibin YAO[4]

1. *School of Geosciences and Info-physics*, *Central South University*, *Changsha* 410012, *China*; 2. *School of Surveying and Geoinformation Engineering*, *East China University of Technology*, *Nanchang* 330000, *China*; 3. *College of Surveying and Geo-informatics*, *Tongji University*, *Shanghai* 200092, *China*; 4. *School of Geodesy and Geomatics*, *Wuhan University*, *Wuhan* 430072, *China*

Abstract: Geodetic functional models, stochastic models, and model parameter estimation theory are fundamental for geodetic data processing. In the past five years, through the unremitting efforts of Chinese scholars in the field of geodetic data processing, according to the application and practice of geodesy, they have made significant contributions in the fields of hypothesis testing theory, un-modeled error, outlier detection, and robust estimation, variance component estimation, complex least squares, and ill-posed problems treatment. Many functional models such as the nonlinear adjustment model, EIV model, and mixed additive and multiplicative random error model are also constructed and improved. Geodetic data inversion is an important part of geodetic data processing, and Chinese scholars have done a lot of work in geodetic data inversion in the past five years, such as seismic slide distribution inversion, intelligent inversion algorithm, multi-source data joint inversion, water reserve change and satellite gravity inversion. This paper introduces the achievements of Chinese scholars in the field of geodetic data processing in the past five years, analyzes the methods used by scholars and the problems solved, and looks forward to the unsolved problems in geodetic data processing and the direction that needs further research in the future.

Key words: stochastic model; functional model; robust estimation; variance component estimation; geodetic data inversion

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1 Introduction

Surveying and mapping is one of the most fundamental disciplines. Whether it is the development and protection of land resources, engineering construction, and satellite navigation, it is inseparable from the support of surveying and mapping disciplines[1]. Data processing is an important part of the discipline of surveying and mapping, and the establishment and development of the classical theory of surveying and mapping has promoted its development. As the form, data, and tasks of modern measurement change, measurement data is processed rationally[2]. Geodetic data processing is an important branch of surveying and mapping data processing, and the development of geodesy is inseparable from the support of geodetic data processing, so it is necessary to continuously develop new theories and methods of geodetic data processing to cope with the difficulties and challenges faced by geodesy on the way forward[3].

Multiple alternative hypotheses and ill-posed problems are often required in actual measurement data processing, so the classical hypothesis testing theory on the basis of a single alternative is unable to satisfy the needs of actual measurement data processing[4]. Since the complex spatiotemporal characteristics of several systematic errors of Global Navigation Satellite System (GNSS) signals and the limited understanding of their physical knowledge, un-modeled errors exist objectively and cannot be avoided, and it is necessary to establish un-modeled error theory to enhance the precision of GNSS signals[5]. In robust

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Foundation support: National Natural Science Foundation of China (No. 42174011)
First author: Jianjun ZHU (1962—), male, professor, PhD supervisor, majors in surveying adjustment and InSAR data processing.
E-mail: zjj@csu.edu.cn
Corresponding author: Leyang WANG
E-mail: wleyang@163.com

estimation, the traditional median variance estimation is biased, which is a difficult problem to be solved[6]. Since the maximum likelihood method is not applicable in some random models, it is introduced in the estimate of variance component[7]. In view of the complexity of the traditional approximation function expression method of nonlinear functions, Chinese scholars have constructed a new theory and method of nonlinear adjustment[8]. In order to address ill-posed problems, many novel methods have been introduced to enhance the precision of parameter estimation. On the basis of the Errors-In-Variables (EIV) model, Chinese scholars have carried out different extended studies and improved the theory and method of total least squares[9]. The traditional additive error model is difficult to satisfy the requirements of modern measurement data processing, so the mixed additive and multiplicative random error model has been proposed in the geodetic field, but the research and application of this model are still relatively small[10]. Modern geodetic observations are not only real numbers, but in some cases complex numbers, and how to adjust complex data is a great challenge[11].

The traditional geodetic inversion method to determine the smoothing coefficient has problems such as excessive dependence on data fitting, low computational efficiency, and low accuracy[8]. Recently, the hot intelligent algorithm is also widely applied in earthquake inversion. With the continuous development of modern measurement, there are many techniques for obtaining measurement data[12]. The use of data from different sources for joint adjustment is still an important trend in geodetic inversion, and multi-data joint adjustment methods are also evolving. With the development of modern measurement technology, scholars have also proposed many new data processing models and methods to better estimate the change in Antarctic water storage[13].

In this article, some of the contributions made by Chinese scholars to geodetic data processing in the past five years are presented. Next, this paper mainly introduces two aspects, the first part is the progress of data processing theory and methods; the second part is the progress of geodetic data inversion.

2 Data Processing Method

2.1 Hypothesis testing theory, unmodeled error and robust estimation problems

The classical hypothesis testing and reliability theory is only based on a single alternative hypothesis, which is inconsistent with the practices. Literature [4] first expanded the primal outlier separability theory based on two optional assumptions to the general situation in which there were several optional assumptions and then proposed the new internal reliability index-Minimal Separability Bias (MSB) that controls both the probabilistic of missed checks and false exclusion[14]. Recently, Literature [15] further formulated the complete quality control indices and their algebraic estimation method for the unifying Detection, Identification and Adaption (DIA) estimators, by considering the uncertainty of the ensemble evaluation-test program with uncertainty propagation. Literature [16] provided an extension of the theory of assumption test to the ill-posed model, in which the total testing statistics, w-test statistics, and the minimum detectable deviations are all reformulated. Then, the deviation-corrected w-test and overall-test statistics were developed for the approximation of a standard normal distribution and two non-centered chi-square distributions, separately.

In addition to the outlier, there is another important source of abnormal error, i.e., unmodeled error. Given the systematic errors such as the spatiotemporal complexity and limited knowledge of GNSS signals, some remnant observation errors may not be removed or alleviated through differences and combinations of observations, model revision, and model parameterization[5,17]. Literature [5] first suggested a program based on the combinatorial use of hypothesis testing to test the significance of the un-modeled error. Literature [17] presented a real-time site-specific un-modeled error detection strategy by using dual-band C/N0. Based on this, stochastic model compensation approaches like the composite model[18] and functional model compensation approaches like multi-epoch partial parameterization[19] were proposed. The results show that theories and methods for dealing with un-modeled errors can significantly further enhance the accuracy and stability of GNSS positioning and navigation.

For robust estimation problems, Literature [6] first proved that the conventional median variance estimation is biased in the case where the number of samples is relatively small, and put forward an unbiased median variance estimation for correcting the deviation in variance estimation. On the basis of the unbiased median variance estimation, the M-estimator with Institute of Geodesy & Geophsics (IGG) III decrease factor was constructed to reduce the bias induced due to the variance estimation. Literature [20] first derived the weighted least-squares algorithm to solve the partial errors-in-variables model and also presented the novel relevant observation RWTLS approach that resolves the original value of robust iterations with the median-parameter algorithm. Based on the above, it was proposed a single novel robust estimation approach on the basis of standardized remnant errors that take into account the effect of the total error on the observation and the structure space at the same time. Literature [21] developed a robust M estimate algorithm that is applicable to three-dimensional (3D) correlation vector observations. An improved IGGIII two-factor shrinkage model was developed, in which the weight reduction factor for three-dimensional vector observations is given with a newly developed test statistic that is coincident with the estimation direction of outlier vectors. Using the presented two-factor reduced model, outlier vectors of observations are reduced directly in the particular direction instead of being reduced on each of the three components respectively. Literature [22] presented a new robust Total Least Trimmed Squares (TLTS) estimator without exhaustive evaluation for the EIV model, however, some block structure is required for the covariate matrix of the independent variables. Literature [23] introduced the robust estimate to the total least-squares method and developed a novel robust weighted total least-squares method to eliminate outliers.

2.2 Variance component estimation and stochastic model

For Variance Component Estimation (VCE) problems, Literature [24] introduced the principle of variable projection and derived replacement formulae of the structure EIV model through the application of Lagrange multipliers. Then, they applied Least-Squares Variance Component Estimation (LS-VCE) to make estimates of various (co-) variance components in the structure EIV model. Literature [25] developed a VCE approach called the least-square variance-covariance component estimation approach on the basis of the Equivalent Conditional Misclosure (LSV-ECM). Literature [7] proved that the maximum likelihood algorithm is not useful for estimating the variance components of certain stochastic models in conventional measurement systems for which the likelihood function is unbounded under certain conditions. In order to enhance further the qualitative aspects of the estimates that are on the basis of the variance components estimate within the partial EIV model, Literature [26] introduced a jackknife resampling algorithm to perform the deviation calculation and correction of bias. Literature [27] proposed the total solution on the basis of the EIV model, taking into account the observation errors for all variables. The variance-covariance component estimation algorithm was introduced into the solution for the regression prediction model to correct the prior cofactor matrix of the random model and the non-common points that were to be predicted. To address the question of estimating the variance components with large-scale observations, Literature [28] proposed a highly efficient VCE method with strict trace computation on the basis of a local-global parameter partitioning scheme in satellite geodesy, and the algorithm is straightforwardly suitable to the simple, but usual, case in which the local parameter is unique to a single set of observations, as well as for the generalized case in which the local parameter is common to various sets of observations.

2.3 Nonlinear accuracy assessment

For the accuracy analysis of nonlinear function problems, Wang and his research group systematically researched the theories and methods of nonlinear adjustment precision estimate on the basis of Sigma point. From the perspective of the probability distribution of approximate function, Wang and his research group proposed the unscented transformation algorithm on the basis of deterministic Sigma points[8], the Sterling interpolation algorithm on the basis of deterministic Sigma points[29-30], the adaptive Monte Carlo algorithm on the basis of random Sigma points[31-32], Jackknife algorithm on the basis of resampled Sigma points[33], and Bootstrap method based on resampled Sigma points[34]. These new theories and methods of nonlinear adjustment precision estimation are constructed to solve the problem that the traditional approximate function expression method relies on complex derivative operations. These theories and methods have been applied and proved in seismic fault parameter inversion, volcano Mogi model parameter inversion, volcano CDM model parameter inversion, GNSS baseline vector solution, digital elevation model, coordinate conversion, forward intersection, Gauss projection coordinate forward calculation, satellite clock error prediction, spatial straight line fitting, ellipse fitting and other measurement data processing fields. These researches are of great significance in both theory and application, which further improves and develops the geodetic data processing theory. In addition to this, Literature [35] proposed an adaptive relaxation algorithm for stable nonlinear parameter estimation on the basis of regularization methods. Literature [36] proposed a New First-Order Approximate (NFOA) precision estimation method for assessing the posterior accuracy of the weighted total least-squares estimation for the EIV model. For the particular structure of linear combinations of nonlinear functions in the field of surveying and mapping, an algorithm for separate non-linear least squares on the basis of the Moore-Penrose generalized inverse matrix and entity matrix is presented.

2.4 Ill-posed problems treatment

For ill-posed problems, Literature [37] proposed a new iterative method and proved its convergence for the rank defect adjustment model that has inequality constraints. Literature [38] suggested a virtual observation approach for solving the ill-posed problem of the PEIV model and an accurate estimation method on the basis of the second-order derivative approximate function approach. According to the Mean Square Error (MSE) criterion, a regularization algorithm decreases variances of the estimated parameters of ill-posed models by introducing biases. The uncertainty adjustment model is more severely affected by coefficient matrix and observation errors when it is an ill-posed model. Literature [39] applied the ridge estimation algorithm to the ill-posed uncertainty adjustment model and derived the iterative approach for improving result stability and reliability. To reduce the biases, Literature [40] proposed a bias reduction method. In this work, they proposed a regularization method to reduce bias in order to enhance a regularization model's MSE for parameter estimation. Truncated Singular Value Decomposition (TSVD) is a valid approach frequently applied to solve ill-posed geodetic questions. Literature [41] performed continuous intercepts of small chi-square values on the basis of the TSVD to obtain variations in both variances and parameter estimates and analyzed these variations to determine the effect on the bias, thus avoiding computational bias when using the parameter's true value. Therefore, the truncation parameters can be identified by the minimum mean square error theory. For the resulting uniqueness and stability, Literature [42] developed a functional model for solving the inequality constraints and proposed to solve the rank-deficit problem using the descending interior point algorithm for potential functions which is based on linear complementarity theory. Literature [43] presented a new ridge estimation approach to address the rank-deficient least squares problem, where rank-deficient matrices are considered to be approximate rank-deficient matrices. They provided an algebraic derivation which means that optimal solutions may actually be acquired through the solution of the associated regularization question for the optimal worst-case residual. Then, they proved the convergence of a novel iterative approach for solving the ridged parameter.

2.5 Total least squares

For the total least squares problems, Literature [9] deduced the formulae to solve the parameter of the

Multivariate Errors-In-Variables (MEIV) model according to the principle of maximum likelihood estimation and proposed two iterative algorithms. Literature [44] proposed an Inequality Constrained Total Least-Squares (ICTLS) scheme that is built on observations directly with constraints, and it was possible to limit independent and dependent variables. Literatures [45]—[47] improved the estimation of the non-negative least squares variance component of the PEIV model to solve the problem of biased variance. Literature [48] proposes a PEIV model correction algorithm based on inequality constraints. Utilizing statistical properties of observation error and coefficient matrix error, Literature [49] derived a new calculation formula for WTLS estimation of the PEIV model on the basis of the Fisher Score method. Literature [50] investigated the number of conditions for the minimum Frobenius criterion for solving multidimensional TLS problems and provided a tight and computable upper-bound estimation method. Literature [51] proposed the Frozen-Barycentre iteration method, which applies Samaski to the barycenter iteration method, realized the conversion of internal iteration and external iteration, saves operation time by reducing derivative calculation, and improved the convergence efficiency of the barycenter iteration method. According to the structural features of the EIV model, Literature [52] divided the design matrix into constant matrix and random matrices, and then reformulated the EIV model into a general structure model and reformulated it into the effective WTLS method, where the algorithm appends merely the weight matrix to the random matrix to decrease the matrix size involving in the iterative procedure. Literature [53] incorporated the second-order term into the constant term of the EIV model, thus representing the nonlinear general EIV model to be a linear Gauss Helmert model, deduced the linearized total least squares algorithm and approximate precision estimation formula of the general EIV model. Literature [54] put orthogonal geometric information as constraint conditions into weighted total least squares and proposed a non-linear equality-constrained total least squares adjustment method with unknown corrected errors for the constraint function. Based on the EIV model, Literature [55] constructed an affine function by using the augmented matrix that consisted of coefficient matrices and observation vectors, reconstruct the function model by using the variable projection method, and deduce a minimum double estimation algorithm for the structure population by using the Lagrangian method. Literature [56] introduced the uncertain total least squares estimate for the linear regression model on the basis of the least squares estimate and uncertainty theory. Based on the equality-constrained non-linear Gauss-Helmert (GH) model, Literature [57] used the Euler-Lagrange method to obtain the least squares solution of a non-linear GH model with equivalent constraints, and then expressed it in a standard constrained least squares problem, introducing a unique sensitivity analysis method. Under the total least squares criterion, Literature [58] transformed the calculation of inequality-constrained total least squares solution into a quadratic programming problem according to the Kuhn-Tucker condition of the optimal solution and proposed an improved Jacobian iterative method to solve the quadratic programming problem. Literature [59] proposed the optimality conditions for Inequality-Constrained Weighted Total Least Squares (ICWTLS) solution in inequality constrained PEIV model. They then revised the current Linear Approximation (LA) method to make it applicable to cross-correlated data. According to the optimality conditions, the Sequential Quadratic Programming (SQP) method is presented. Due to the fact that the Hessian matrix is hard to calculate in the SQP method and the convergence is slow or even non-convergent while the Hessian matrix is an indefinite positive matrix, the Damped Quasi-Newton (DQN) SQP approach is introduced. Taking into account the coordinate errors of control points and non-control points, and different weights among them, Literature [60] put forward an iterative method to extend Weighted Total Least Squares (WTLS) 3D similarity transformation on the basis of Gibbs vector.

2.6 Mixed additive and multiplicative random error model

For the multiplicative error model and mixed additive and multiplicative random errors model, Literatures [61]—[62] obtained the second-order accuracy information of the parameters of the multiplicative error model and the mixed additive and multiplicative error random model by using the Sterling method and SUT method without derivative calculation, respectively. For the ill-posed multiplicative error model, Literature [6] proposed to use the A-optimal design algorithm to identify the regularized parameters for the ill-posed model and established a more reliable parameter estimation method by combining it with the iterative method of virtual observation. For the mixed additive and multiplicative errors model with constraints, Literature [63] constructed an exhaustive method for parameter estimation of the ill-posed model under inequality constraints and proved the effectiveness of this method to deal with related problems. Literature [64] extended the mixed additive and multiplicative errors model to a more general generalized mixed additive and multiplicative error model with a deterministic trend. Literatures [65]—[66] applied the cat colony algorithm and artificial bee colony algorithm to the parameter estimation of the mixed additive and multiplicative random error model. Literature [67] proposed an inequality constraint algorithm for the mixed additive and multiplicative random error model, which complemented the model theory.

2.7 Complex least squares

For most of the geodetic applications, the observables are real numbers and least squares-based adjustment has been widely used to reduce the error and obtain the optimal estimation. However, the observable of InSAR is recorded by a complex number which is related to the interferometric phase and correlation. In such a case, complex least squares-based adjustment is needed. To develop the complex least squares-based adjustment, the critical problem is how to establish the least squares criteria and stochastic model for the complex number. Literature [11] proposed a non-linear least squares adjustment method whose least squares criteria is the sum of squares of the module of the residual of complex observations is minimal. With this criterion, the adjustment should be conducted in a complex domain. Literature [68] proposed that the complex observation can be divided into real and imaginary parts and complex least squares adjustment is then a joint adjustment of real and real and imaginary. As a result, the existing adjustment theory for real numbers can also be used to process the complex observable. For the stochastic model, Literatures [68] suggested that the standard deviation of the module of the complex can be regarded as the weight of the complex observable.

3 Geodetic Inversion

3.1 Coseismic slip distribution inversion

The geodetic inversion methods were further developed. To determine the smooth coefficient (also called the regularization parameter) in coseismic slip distribution inversion, it was common to use a compromised curve between the model roughness and the data fitting residuals[69]. On the basis of the L-curve, the Eclectic Intersection curve was proposed which is a new method for determining the smoothing coefficients by Literature [8]. In contrast to the L-curve algorithm, the Eclectic Intersection curve method has advantages such as high computational efficiency, independence from data fitting, and more suitable smooth coefficients. In Literature [70], the properties of high-rate single GNSS and multi-GNSS fusion in early warning magnitude computation, fast mass moment tensor inversion, and static fault slip inversion was thoroughly studied using data related to the *Mw* 7.4 magnitude earthquake in Mado County, Qinghai Province, China. Literature [71] suggested a new method for determining coseismic slip distribution inversion for multi-observation types, that allows for the simultaneous determination of relative weighting ratios and regularization parameters of multiple observations.

3.2 Intelligent inversion algorithm

At the same time, intelligent algorithms have been widely used in seismic inversion problems. Literature [12] presented a Bayesian estimation-based method for fault parameter inversion, which can quantify the optimal model parameter uncertainty through the posterior probability distribution in the parameter space. Literature [72] suggested a fault parameter inversion algorithm which is based on the combination algorithm of a genetic algorithm and an iterative least squares algorithm, which can give consideration to the sensitivity and correlation of fault parameters. Literature [73] introduced a scale-free trace transformation algorithm using a determined sample strategy for non-linear inversion and precision estimation of the seismic fault parameter. Literature [74] proposed an Adaptive Multi-start Gauss-Newton Algorithm (AMGNA) to invert seismic source parameters using multiple geodetic datasets. Literature [75] proposed a Particle Swarm Optimization Algorithm (BH-PSO) incorporating a black hole strategy. Compared to the Simulated Annealing (SA) method, the BH-PSO method is less time-consuming and more accurate than the Genetic Algorithm (GA). Traditional Genetic Algorithm (GA) inversion results are unstable and easily fall into local optimum in an inversion of fault parameters[76]. Literature [76] proposed a Genetic Nelder-Mead Neural Network Algorithm (GNMNNA). GNMNNA outperforms GA and NNA in terms of inversion accuracy and calculation stability, and GNMNNA provides more potential for application under more complicated seismic environments. Literature [77] introduced a general variational inference algorithm, Automatic Differential Variational Inference (ADVI) to Bayesian slip inversion, and compared it to the classical Metropolis-Hastings (MH) sampling algorithm. Literature [78] proposed a deep learning method (referred to as ESPI-ResNet) for the inversion of seismic source parameters on satellite InSAR data using ResNet. Literature [79] introduced the Sterling interpolation method to evaluate the precision of parametric non-linear inversions and applied it to the Lushan and L'Aquila earthquakes.

3.3 Multi-data joint inversion

Multi-data joint inversion is still the development trend of geodetic inversion. Interferometric Synthetic Aperture Radar (InSAR) has emerged as a significant technique to study seismic cyclic deformation. In order to obtain completive and precise three-dimensional (3-D) displacements of the ground surface from heterogenous InSAR displacement observations, Literature [80] proposed an approach on the basis of Strain Model and Variance Component Estimate (SM-VCE), in which the Robust Variance Component Estimation (RVCE) algorithm was utilized to weight different InSAR observations. It has been proved that the SM-VCE method is obviously superior to the traditional InSAR empirical weighting method for obtaining 3-D displacements. In addition to the co-event 3-D displacements, Literature [81] introduced a novel Kalman Filter-based InSAR method (KFInSAR) to combine multiple InSAR time series observations to evaluate the time-series 3-D displacements, where each InSAR time series dataset is optimized with an Iterative Weighted Least Square (IWLS)-based error correction procedure[82], and the decorrelation noise and atmospheric delay can be significantly decreased. Literature [83] used interferometric synthetic aperture radar data to analyze the coseismic and postseismic displacement fields in association with the 2016 Central Petermann Ranges earthquake in Australia. Literature [84] developed a Logarithmic model-based approach (LogSIM) to jointly invert co-seismic and post-earthquake fault sliding with InSAR data of multiple platforms from various tracks. Literature [85] used GPS and InSAR co-seismic deformation fields to co-inverse the sliding distribution model in the Koshien earthquake and analyzed the relationship between the Koshien earthquake and the Mino earthquake according to static Coulomb stress alteration in conjunction with previous research results, and also constructed a fault grid for seven major faults in southwestern Taiwan and obtained their stress alteration models. Literature [86] used rising and falling Sentinel-1 Interferometric Synthetic Aperture Radar (InSAR) images to build coseismic displacements related to the *Mw* 7.1 Anchorage earthquake that showed a subcircular deformation pattern of -4 cm of subsidence in the sight-line direction. Literature [87] used three-dimensional coseismic displacement fields from spatial imaging geodesy to invert the *Mw* 7.8 Kaikoura, New Zealand earthquake. Literature [88] provided a new explanation for the 2016 Mino earthquake from Synthetic Aperture Radar (SAR) satellite, high-speed GPS, and strong motion data. Literature [89] proposed the improved Spatio Temporal Random Effects (STRE) model and the Multire Solution Segmentation Fusion (MRSF) method for InSAR and GNSS fusion, which can better reveal the spatial heterogeneity and the slip distribution of deformation data in Cascadia Subduction Zone and San Francisco Bay region, California. Literatures [90]—[92] estimated the 3-D co-seismic displacements of the 6th July 2019 Ridgecrest earthquake, California, the 22nd May 2021 Maduo earthquake, the 9th January 2022 Menyuan earthquake, China, etc. from InSAR and pixel-offset tracking observations based on the SM-VCE method, providing a valuable and precise dataset to constrain the movement of faults. Literatures [11] and [68] estimated the forest height and sub-canopy topography with the complex least squares adjustment method from polarimetric InSAR data.

3.4 Water storage variation and satellite gravity inversion

Antarctic Basal Water Storage Variation (BWSV) is the mass variation of liquid water under the Antarctic ice cap[12]. Literature [12] proposed a stratified gravity density forward/inverse Antarctic BWSV estimation method and associated model based on multi-source satellite observation data. Numerous recent mass balances with the Gravity Recovery And Climate Experiment (GRACE) and satellite height (among both radar and laser sensors) have used a large number of forward models with uncertainties[86]. To minimize the considerable sources of error associated with the use of forward models, Literature [93] used multiple geodetic observations to estimate mass changes in the ice cap and present-day Glacial Isostatic Adjustment (GIA), consisting of GRACE and the Ice, Cloud, and Land Elevation Satellite (ICESat), and the Global Positioning System (GPS), using the modified Joint Inverse Estimation (JIE) method for simultaneously solving Antarctic GIA and ice trends simultaneously.

Literature [94] analyzed land water reserves using GRACE time-varying gravity field data inversion and periodic characteristics of GLDAS and CPC results. Using the Yellow River Basin as a research region to obtain the deficient equivalent water height, it was found that the overall water storage in the research region decreased with a speed of -0.51 ± 0.03 cm/a from 2002 to 2020. The lag time of equivalent water height retrieved by GRACE is 2 ~ 3 months with respect to precipitation, evapotranspiration, and soil temperature. It was verified by experiments that GRACE has a strong correlation with equivalent water height calculated by GLDAS and CPC, and both have obvious annual resonance periods.

4 Summary and Outlook

4.1 Summary

Following the evolution of electronic information technology and the changes in the needs of engineering projects, data acquisition methods, data categories, processing tasks, and other aspects have undergone tremendous changes. During the last five years, Chinese researchers have greatly promoted the development of surveying and mapping data processing. Combined with practical applications, Chinese scholars have developed a number of alternative hypothesis theories, and at the same time applied hypothesis testing theories to the treatment of ill-posed problems and outlier treatment, which improved the accuracy of ill-posed problem and outlier handling. Aiming at the unmodeled error in GNSS, a combined algorithm based on a construction test is proposed to compensate for the model. The traditional median variance estimation in robust estimation is biased, and Chinese scholars have proposed robust M estimation methods such as TLTS and three-dimensional correlation vector observation to eliminate the bias. On the basis of the EIV model, considering the complexity of the actual measurement data, a variety of new parameter estimation methods, inequality constraint algorithms, and improved variance component estimation algorithms are proposed. The research on additive multiplicative mixed error model has also made great progress in solving the pathological problem of the model, and also using the intelligent search method for estimating the parameters in the model. A variety of methods are proposed for the accuracy evaluation of nonlinear functions based on SIMGA points, and all of them have good accuracy results. Meanwhile, the least squares criterion is also suitable for the calculation of complex adjustment, which solves the adjustment problem that InSAR data is generated by complex number records.

In terms of geodetic inversion, Chinese scholars proposed new algorithms such as compromise intersection curves to determine the smoothing coefficient on the basis of L-curves. In recent years, popular intelligent algorithms have also been widely used in geodetic inversion, and have achieved good accuracy in actual seismic inversion. With the increasing availability of geodetic data acquisition technology, multi-source data joint inversion is still the focus of research, and Chinese scholars have proposed related joint inversion algorithms according to different data. In the inversion of Antarctic basal water reserves, Chinese scholars make full use of gravity data, climate data, and satellite altimetry data to jointly invert, and improve the relevant inversion algorithms.

4.2 Outlook

Although geodetic data processing has achieved good results, some problems have not been solved, which is also a direction for future research by experts and scholars, some of which are as follows:

(1) It is generally the case that the control of performances of the DIA estimator for the misspecification of medium sizes in weak geometries will represent the most difficult task[15].

(2) How to port the prior model algorithm to the current GNSS positioning software[17]?

(3) Efficient VCE algorithms have better as well as at least equivalent computing efficiency in satellite geodesy but with rigorous trajectory computation results. However, it is worth noting that the implementation of efficient methods for very hyperscale problems, in which high-performance computation and distributed CPU memory environments are critical, requires further research[28].

(4) The existing resampling methods are suitable for heteroscedasticity and independent problems of observation data; Where relevant, the suitability of the resampling method for variance component estimation requires further investigation[33].

(5) The quasi-Monte Carlo method has rich theoretical achievements, and how to further develop the relevant algorithms, expand the types of approximate probability density distribution methods, and apply them to geodetic data processing needs further research[32].

(6) Ridge estimation in biased estimation is a particular instance of regularization methods. Ridge estimation is a special case of generalized ridge estimation, so the regularization method is more important, and how to find a method for determining regularization parameters quickly, accurately, and effectively remains to be studied[33].

(7) In complex least squares, more precise model constraints and time decorrelation will be taken into account to enhance the suitability of the inversion algorithm. The prior correlation conditions of random elements of the observation vectors and coefficient matrices will be studied[11,60].

Functional models are the basis of geodetic data, and there are still many problems that remain unsolved in various models and geodetic inversions, such as:

(1) The accuracy loss range and influencing factors of the nonlinear EIV model to the linear model need to be studied, and the cut-off threshold for truncating singular values and the selection of correction locations for correcting singular values need to be further studied. The accuracy of the various solutions is evaluated singly, and only the approximate first-order methods-variance and covariance are used[31].

(2) How to apply the regularization method in joint adjustment (taking into account relative weights) remains to be studied, how to extend the accuracy evaluation method to the second-order method is yet to be studied, and the accuracy evaluation of stochastic models of EIV joint models has not been studied in depth, and the deviation of parameter estimates is calculated.

(3) The variance component estimation, correlation problems and practical application of mixed

additive and multiplicative random error model all need to be studied.

(4) On the basis of the rectangular dislocation theory, it is difficult to achieve continuous smooth simulated surface faults by dividing the fault plane into sub-faults, and how to set the sub-segment level needs further research.

(5) While identifying the relative weight ratio for the joint inversion, there are many methods, and each method is based on a certain criterion, how to analyze the advantages of various methods, and propose a generality determination method needs further research. In multi-source data joint inversion, the difference between different data is not only in terms of accuracy, there are differences in the number, spatial distribution and time span of data, and how to consider various differences and quantify the differences between them needs further research.

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