测绘出版社
SURVEYING AND MAPPING PRESS

Approximately Conformal, Equivalent and Equidistant Map Projections

Miljenko LAPAINE, Nedjeljko FRANČULA

Faculty of Geodesy, University of Zagreb, Zagreb 10000, Croatia

Abstract: If geodetic coordinates from an ellipsoid are included in the equations of a projection for mapping a sphere instead of geographical coordinates, the result will be a projection of the ellipsoid into a plane. This will slightly change the distortion distribution of the initial map projection. The question is to what extent the replacement of geographical with geodetic coordinates will affect this change. In this paper, we deal with conformal, equal-area and equidistant projections of the sphere, which we modify by using geodetic coordinates instead of geographical ones. The result will be an approximately conformal, approximately equal-area and approximately equidistant projection. It is shown that in this case the maximum distortion of the angles in approximately conformal projections will be approximately 23.09′, the maximum distortion of the area in approximately equal-area projections less than 0.7% and the maximum distortion of lengths in approximately equidistant projections less than 0.7%, therefore on the maps imperceptible.

Key words: map projections; double mapping; approximately conformal projection; approximately equal-area projection; approximately equidistant projection

Citation: Miljenko LAPAINE, Nedjeljko FRANČULA. Approximately Conformal, Equivalent and Equidistant Map Projections [J]. Journal of Geodesy and Geoinformation Science, 2022, 5(3): 33-40. DOI:10.11947/j.JGGS.2022.0304.

1 Introduction

A map is a projection of data usually from the real Earth, celestial body or imagined world to a plane representation on a piece of paper or on a digital display such as a computer monitor. Usually, maps are created by transforming data from the real world to a spherical or ellipsoidal surface (the generating globe) and then to a plane. The characteristics of this generating globe are that angles, distances or surfaces measured on it are proportional to those measured on the real Earth[1-2]. The transformation from the curved surface into a plane is known as map projection and can take a variety of forms, all of which involve distortion of areas, angles, and/or distances. Since no map projection maintains the correct scale throughout, it is important to determine the extent to which it varies on a map. On a world map, qualitative distortion is evident to an eye familiar with maps, after noting the extent to which landmasses are improperly sized or out of shape, and the extent to which meridians and parallels do not intersect at right angles or are not spaced uniformly along a given meridian or given parallel. On maps of countries or even of continents, distortion may not be evident to the eye, but it becomes apparent upon careful measurement and analysis[3-5].

All map projections involve distortion of areas, angles, and/or distances. The types of distortion can be controlled to preserve specific characteristics, but map projections must distort other characteristics of the represented object. The main problem in cartography is that it is not possible to map a spherical or ellipsoidal

Received date: 2022-03-08; accepted date: 2022-08-25
First author: Miljenko LAPAINE, professor, interests in map projections.
E-mail: mlapaine@geof.hr

surface into a plane without distortions. Euler first proved as early as 1772 that a sphere cannot be mapped into a plane with zero-distortion[6-7].

Beginning in the late 1950s, a learned journal Acta Geodaetica et Cartographica Sinica, published by the Chinese Society of Surveying and Mapping started to carry papers on the subject of map projections. The other journals, such as the Bulletin of Surveying and Mapping and Translations of Surveying and Mapping, often published papers about map projections. To meet the requirements of teaching and production, many educational materials were edited and published by Chinese colleges and universities of surveying and mapping. More of the advancement in map projection study in China one can learn from the book by Qihe Yang, John P. Snyder and Waldo R. Tobler[8].

In the cartographic literature, map projections are divided by types of distortion into conformal, equidistant (in a certain direction), equal-area and compromise projections[9]. In the Russian cartographic literature, projections with small distortion of angles and projections with small distortion of area have also been added. These are projections in which angle distortions or area distortions are not noticeable to the naked eye[10]. What distortions of lengths, shapes and area are not noticeable on the maps, Ginzburg[11] announced on the basis of his experimental research. He came across this information:

(1) With winding lines such as rivers, coastlines and boundaries, differences in length of up to 5% cannot be observed. Only differences of 10% and higher are easy to spot.

(2) When the boundary of an area is a curved line, differences of only 5% become noticeable. Differences of 10%~15% are easily noticeable.

(3) Shape distortions occur already at deformations of angles of 2°~3°. Distortion of angles of 4°~5° causes easily noticeable distortion of the shape.

The need for approximately conformal and approximately equal-area projections was explained in the Russian cartographic literature of the 1950s by these arguments. Since general geographical atlases and most of the various hand maps are intended for a wide range of users, most of whom do not know the methodology of measurement on maps, the visual interpretation of cartographic content is much more important. Therefore, it is necessary for the cartographer to know the possibilities of visual interpretation to be able to use them in making the mathematical basis of maps[10].

Bugayevskiy and Snyder[12] when choosing map projection divide all maps into technical and general, and accordingly distinguish the method of perception and evaluation of cartographic information. The application of the data given in their Appendix 1 enables the experience of the magnitude of the distortions that can be neglected, the development of an approximate hierarchy of requirements for map projections and the understanding of the nature of the distortions of the projection and scale. They distinguish the limit values of distortions whose effect can still be neglected depending on whether they are scientific-technical and technical maps or maps for general use.

If it is a question of scientific-technical or technical maps and analysis of cartographic information mainlybased on cartography of higher accuracy and with less use of computers, then distortion of linear scale and area scale up to ± (0.2%~0.4%) and angles up to 15′~30′ can be tolerated.

Maps for general use which are used for analyzing and using cartographic information by rough measurements and estimating the dimensions, shape, relative position and significance of the area, distortions of length and area up to ± (2%~3%) and angles up to 2°~3° are allowed. If cartographic information is determined and estimated mainly visually, sometimes by rough measurements, as is the case with wall maps, some maps in atlases and textbooks, and maps for illustration in various publications, then distortions of length and area up to ± (6% ~ 8%) may be allowed, and angles up to 6° ~ 8°.

Such distortions cannot be visually observed. If cartographic information is observed and assessed only visually and without measurement, e.g., from wall maps, school maps or illustrative maps, then distortions of length and area up to ± (10% ~ 12%) and angles up to 10° ~ 12° can be tolerated. In doing so, some deformations will be visible.

Today, when most of the data we use in making maps is in digital form, or easily digitized into digital form, the application of approximately conformal and approximately equal-area projections is even more justified. For example, a map in equal-area projection is not necessary to determine areas. The area can also be determined according to the data obtained from the maps in conformal projections if the distortions of the projection are considered. In this way, as early as 1993, the area of the Croatian sea and islands was determined from the map at a scale of 1 : 1 000 000 in the Gauss-Krüger, i.e., conformal, projection[13]. Using this methodology, which was elaborated in several other articles[14-16], determined the lengths of coastlines and areas of Croatian islands from a topographic map at a scale of 1 : 25 000 produced in the Gauss-Krüger projection.

Another possibility of determining lengths and areas is to calculate the geodetic coordinates on the ellipsoid from the coordinates in a map projection and to calculate the lengths and areas on the ellipsoid.

And what should be especially emphasized, today many online cartographic services, such as Google Maps and virtual globes such as Google Earth, contain options for measuring the lengths and areas free from the effects of projection distortion[17].

2 Double Mapping of an Ellipsoid into Plane

An ellipsoid can be mapped to a plane in two ways: ① directly, i. e., express the coordinates of an arbitrary point in the plane directly using geodetic coordinates, or ② first map the ellipsoid to a sphere and then map that sphere to the plane. The second approach is called the double mapping. The meaning of this second approach is important for cartography, because only in this way it is possible to calculate oblique map projections of an ellipsoid using the transition to normal one by spherical trigonometry and at the same time consider the Earth's flatness[18]. Formulas that connect the equations of normal, transverse, and oblique projections are given by Lapaine and Frančula[19].

For example, if an ellipsoid is conformally mapped to a sphere[20-21], and then that sphere is conformally mapped to the plane using one of the conformal projections in the oblique aspect, the corresponding conformal projection of the ellipsoid is obtained-stereographic oblique, Mercator oblique, etc. The same is true for equal-area mapping, i.e., if the mapping of an ellipsoid to a sphere is equal-area[22] and mapping a sphere into a plane is equal-area, then the composition of these mappings is equal-area to mapping the ellipsoid into a plane.

As for the property of equidistance, the analogous assertion is generally not true. Only in the case when the main directions, along which the main scale is equal to one, were the same during equidistant mapping of the ellipsoid onto the sphere and the equidistant mapping of the sphere onto the plane, it is possible to conclude that the equidistant projection of the ellipsoid into the plane will result[23].

The process that Google used in 2005 to create the mathematical basis for its online mapping service Google Maps can be interpreted as the double mapping of an ellipsoid into a plane[24]. However, the ellipsoid is not mapped to the sphere either conformally[20-21] or equal-areat[22] or equidistant[23], but provided that the geographical coordinates on the sphere are equal to the geodetic coordinates on the ellipsoid. In this paper it is described by Eq. (2). The sphere is then mapped into a plane according to the formulas of the normal conformal cylindrical projection, i.e., Mercator projection. For the radius of the sphere, the large semiaxis of the ellipsoid WGS84 was taken. WGS84 is widely used today all over the world. In this paper it is described by Eq. (1). Since the ellipsoid is not conformally mapped to the sphere, the resulting projection is not a conformal mapping of the ellipsoid into a plane, but approximately conformally. The newly obtained projection is known as the web-Mercator projection.

It is important to emphasize once again that the mapping of the ellipsoid into the plane as applied by Google, in practice is achieved by including geodetic coordinates from the ellipsoid in the formulas for the mapping sphere, so there was no additional calculation. The main reason for Google's application of such a procedure is simpler formulas for mapping a sphere and therefore faster calculations than direct formulas for mapping ellipsoid[25].

2.1 Scales of mapping of an ellipsoid to a sphere by normals

To calculate the distortions in an approximately conformal projection, it is necessary to have formulas for the factors of the local linear scale in the direction of the meridians and parallels when mapping the ellipsoid to the sphere and then the sphere to the plane.

We will call the latitude and longitude related to the rotating ellipsoid geodetic coordinates and denote φ, λ. We will call the latitude and longitude related to the sphere by geographical coordinates and denote φ', λ'.

Let a rotating ellipsoid with semiaxes a and b be mapped to a sphere of radius R so that

$$R=a \tag{1}$$

$$\varphi'=\varphi,\ \lambda'=\lambda \tag{2}$$

The mapping of an ellipsoid to a sphere (Fig.1) given by Eq. (2) is known in the literature as mapping by normals[18].

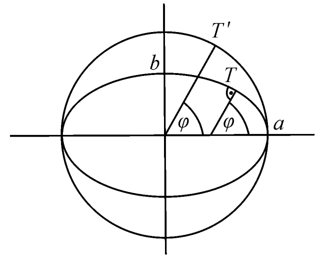

Fig. 1 Mapping an ellipsoid to a sphere by normals (Point T' is the image of point T)

Factors of local linear scales by meridians h_1 and by parallels k_1 of mapping given by Eqs. (1) and (2) are

$$h_1=\frac{R\mathrm{d}\varphi'}{M\mathrm{d}\varphi}=\frac{a}{M}=\frac{\sqrt{(1-e^2\sin^2\varphi)^3}}{1-e^2} \tag{3}$$

$$k_1=\frac{R\cos\varphi'\mathrm{d}\lambda'}{N\cos\varphi\mathrm{d}\lambda}=\frac{a}{N}=\sqrt{1-e^2\sin^2\varphi} \tag{4}$$

where, $M=\dfrac{a(1-e^2)}{\sqrt{(1-e^2\sin^2\varphi)^3}}$, the radius of curvature of the meridian; $N=\dfrac{a}{\sqrt{1-e^2\sin^2\varphi}}$, the radius of curvature of the first vertical; and $e=\sqrt{1-\dfrac{b^2}{a^2}}$, the first numerical eccentricity of the ellipse.

Local area scale factor is

$$p_1=h_1k_1=\frac{(1-e^2\sin^2\varphi)^2}{1-e^2} \tag{5}$$

The maximum distortions of the angles ω_1 are calculated

$$\sin\frac{\omega_1}{2}=\frac{|h_1-k_1|}{h_1+k_1}=\frac{e^2\cos^2\varphi}{2(1-e^2)+e^2\cos^2\varphi} \tag{6}$$

Values of h_1, k_1, p_1 and ω_1 with $e^2=0.006\,694\,38$ are represented in Tab.1.

From Tab. 1 when mapping the ellipsoid to the sphere by normals, the distortions of the lengths are not more than 0.7%, the distortions of the area are also not more than 0.7% and the distortions of the angles are not more than 23.09′. These are invisible sizes to the naked eye.

Tab.1 Factors of local linear scale in the direction of meridian h_1, in the direction of parallel k_1, factor of local area scale p_1 and maximum distortion of angle ω_1 as a function of geodetic latitude φ, according to Eq. (3)—Eq. (6) with $e^2 = 0.006\,694\,38$

$\varphi/(°)$	h_1	k_1	p_1	$\omega_1/(')$
0	1.006 739	1	1.006 739	23.090 92
10	1.006 435	0.999 899	1.006 333	22.396 91
20	1.005 557	0.999 608	1.005 163	20.397 80
30	1.004 213	0.999 163	1.003 373	17.332 73
40	1.002 565	0.998 616	1.001 178	13.569 12
50	1.000 813	0.998 034	0.998 845	9.5594 35
60	0.999 167	0.997 486	0.996 656	5.7872 96
70	0.997 826	0.997 040	0.994 872	2.7091 53
80	0.996 951	0.996 748	0.993 709	0.6985 50
85	0.996 724	0.996 673	0.993 407	0.1759 88
90	0.996 647	0.996 647	0.993 306	0

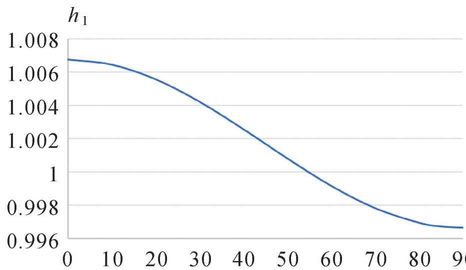

Fig.2 Factor of local linear scale in the direction of meridian h_1 as a function of geodetic latitude φ, according to Eq. (3)

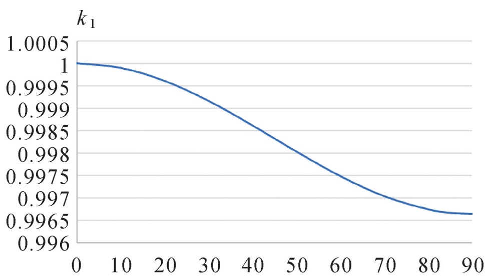

Fig.3 Factor of local linear scale in the direction of meridian k_1 as a function of geodetic latitude φ, according to Eq. (4)

3 Approximately Conformal, Equal-Area and Equidistant Projections of an Ellipsoid onto a Plane

If geodetic coordinates from an ellipsoid are substituted in the equations of a map projection for mapping a sphere, the result will be a projection of the ellipsoid into a plane. Regardless of the projection of the sphere in this mapping composition, the factors of local scales of lengths and areas of such double mapping are obtained by multiplying the corresponding factors of local scales of lengths and areas of projection of the sphere by h_1, k_1 and p_1 according to Eqs. (3), (4) and (5).

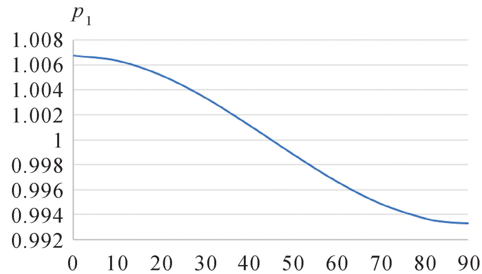

Fig.4 Factor of local area scale p_1 as a function of geodetic latitude φ, according to Eq. (5)

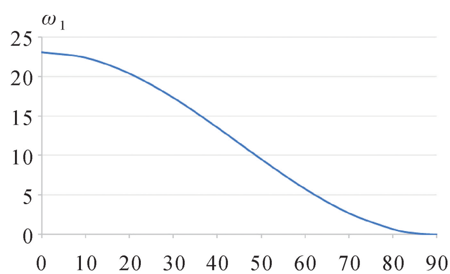

Fig.5 Maximum distortion of angle ω_1 in minutes as a function of geodetic latitude φ, according to Eq. (6)

3.1 Approximately conformal projections of the ellipsoid into the plane

If geodetic coordinates from an ellipsoid are included in the equations of the conformal projection for mapping a sphere, the result will be an approximately conformal projection of the ellipsoid into the plane. Regardless of the conformal projection of the sphere in that mapping composition, the maximum distortion of the angle of such a double mapping is expressed by Eq. (6). Tab.1 gives the values of the maximum distortions of the angle $\omega = \omega_1$ with $e^2 = 0.006\,694\,38$. Fig.5 shows these values as a function of geodetic latitude φ. Eq. (6) can be written in the form

$$\sin\frac{\omega_1}{2}=1-\frac{1}{1+\frac{e^2\cos^2\varphi}{2(1-e^2)}} \tag{7}$$

whence we see that we will obtain the extreme values (maximum and minimum) of the function $\omega=\omega_1$ for $\varphi=0°$ and $\varphi=90°$.

$$\omega_1(0°)=\omega_{1\max}=\frac{e^2}{2-e^2},\quad \omega_1(90°)=\omega_{1\min}=0 \tag{8}$$

From Tab.1 we can see that the maximum distortion of the angles in the approximately conformal cylindrical projection will be 23.09′ and therefore the distortions of the shape on the map made in such a projection will be imperceptible. However, this does not mean that we do not have to take care of other distortions. E.g., if geodetic coordinates are substituted in the equations of the conformal cylindrical projection for mapping the sphere, the result will be an approximately conformal cylindrical projection of the ellipsoid in the plane. Let us look at it in more detail.

Basic cartographic equations and expressions for scales and distortions in a normal aspect conformal cylindrical projection of a sphere are[26]

$$x=R\cos\varphi_0(\lambda-\lambda_0),\ y=R\cos\varphi_0\ln\left(\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right) \tag{9}$$

where, $\varphi\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$; $\lambda\in[-\pi,\pi]$; constants $R>0$; $\varphi_0\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$; and $\lambda_0\in[-\pi,\pi]$.

We can calculate

$$h_2=k_2=\frac{\cos\varphi_0}{\cos\varphi},\ p_2=h_2k_2=\left(\frac{\cos\varphi_0}{\cos\varphi}\right)^2,\ \omega_2=0 \tag{10}$$

In these equations h_2 is the factor of local linear scale along meridians; k_2 factor of local linear scale along parallels; p_2 factor of local area scale; ω_2 maximum angle distortion; and φ_0 given latitude.

If we first map the ellipsoid onto a sphere according to Eqs. (1) and (2), and then the sphere onto the plane by applying Eq. (9), we will obtain an approximately conformal cylindrical projection

$$x=R\cos\varphi_0(\lambda-\lambda_0),\ y=R\cos\varphi_0\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) \tag{11}$$

$$\left.\begin{aligned} h&=h_1h_2=\frac{\sqrt{(1-e^2\sin^2\varphi)^3}}{1-e^2}\frac{\cos\varphi_0}{\cos\varphi}\\ k&=k_1k_2=\sqrt{1-e^2\sin^2\varphi}\frac{\cos\varphi_0}{\cos\varphi}\end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} p&=p_1p_2=\frac{(1-e^2\sin^2\varphi)^2}{1-e^2}\left(\frac{\cos\varphi_0}{\cos\varphi}\right)^2\\ \omega&=\omega_1=2\sin^{-1}\frac{e^2\cos^2\varphi}{2(1-e^2)+e^2\cos^2\varphi}\end{aligned}\right\} \tag{13}$$

where, all distortions of an approximately conformal cylindrical projection can be determined.

3.2 Approximately equal-area projections of the ellipsoid into the plane

If geodetic coordinates from an ellipsoid are substituted in the equations of equal-area projection for mapping a sphere, the result will be an approximately equal-area projection of the ellipsoid into the plane. Regardless of the equal-area projection of the sphere in that mapping composition, the area scale of such a double mapping is expressed by Eq. (5).

Tab.1 shows the values of the local factor of the area scale $p=p_1$ with $e^2=0.006\,694\,38$. Fig.4 shows the local area scale factor $p=p_1$ as a function of geodetic latitude φ.

The extreme values (maximum and minimum) of the function $p=p_1$ will be obtained for $\varphi=0°$ and $\varphi=90°$.

$$\left.\begin{aligned} p_1(0°)&=\omega_{1\max}=\frac{1}{1-e^2}=1+\frac{e^2}{1-e^2}\\ p_1(90°)&=p_{1\min}=1-e^2=1-\frac{e^2}{1-e^2}\end{aligned}\right\} \tag{14}$$

For $e^2=0.006\,694\,38$ we get

$$\frac{e^2}{1-e^2}=0.006\,719 \tag{15}$$

Therefore, if geodetic coordinates are included in the sphere mapping formulas in any equal-area projection, approximately equal-area projections of the ellipsoid will be obtained. Area distortions will be less than 0.7%, and therefore imperceptible on the maps. However, this does not mean that we do not have to take care of other distortions. The distribution of all distortions should be determined in an analogous way to that shown in the section on approximately conformal projections.

3.3 Approximately equidistant projections of the ellipsoid into the plane

If geodetic coordinates from an ellipsoid are substituted in the equations of the equidistant projection for mapping a sphere, the result will be an approximately equidistant projection of the ellipsoid into the plane. In the case of conic and azimuthal projections, a distinction should be made between equidistance along meridians and equidistance along parallels. Tab.1 shows the values of the local linear scale factor in the meridian direction $h=h_1$ and in the parallel direction $k=k_1$ with $e^2=0.006\,694\,38$ for mapping the ellipsoid to the sphere according to the normals. Figs.2 and 3 show the local scale factors $h=h_1$ and $k=k_1$ as a function of geodetic latitude φ. The extreme values (maximum and minimum) of the functions $h=h_1$ and $k=k_1$ will be obtained for $\varphi=0°$ and $\varphi=90°$

$$h_1(0°)=h_{1\max}=\frac{1}{1-e^2}=1.006\,7;$$

$$h_1(90°)=h_{1\min}=\sqrt{1-e^2}=0.996\,6;$$

$$k_1(0°)=k_{1\max}=1;$$

and $k_1(90°)=k_{1\min}=\sqrt{1-e^2}=0.996\,6.$

The local linear scale factor differs by less than 0.7% from the unit in the equidistant line direction. This means that when choosing an approximately equidistant projection of an ellipsoid, the criteria that apply to equidistant projections of a sphere can be used. However, here, as with all other map projections, we must take care of other distortions.

4 Conclusion

A map is a result of mapping data usually from the Earth, celestial body or imagined world to a plane representation on a piece of paper or on a digital display such as a computer monitor. Usually, maps are created by transforming data to a spherical or ellipsoidal surface and then to a plane. The mapping from a curved surface into a plane is known as map projection and can take a variety of forms.

Since no map projection maintains the correct scale throughout, it is important to determine the extent to which it varies on a map. On a world map, qualitative distortion is evident to an eye familiar with maps, after noting the extent to which landmasses are improperly sized or out of shape, and the extent to which meridians and parallels do not intersect at right angles or are not spaced uniformly along a given meridian or given parallel. On maps of countries or even of continents, distortion may not be evident to the eye, but it becomes apparent upon careful measurement and analysis.

There are no map projections that can maintain a perfect scale throughout the entire projected area because they are taking a sphere or ellipsoid and forcing it onto a flat surface. There are four main types of distortion that come from map projections: distance, direction, shape and area. That is why when applying any map projection, we must take care of all the distortions.

If geodetic coordinates from an ellipsoid are included in the equations of conformal, equivalent, or equidistant projections for mapping a sphere instead of geographical coordinates, the result will be approximately conformal, approximately equal-area or approximately equidistant projection of the ellipsoid into the plane. Equidistance along meridians and equidistance along parallels need to be distinguished for conic and azimuthal projections. The question was to what extent the replacement of geographical with geodetic coordinates will affect this change. The paper shows that the maximum distortions of the angles in approximately conformal projections will be approximately 23.09′, the maximum area distortion in approximately equal-area projections less than 0.7% and the maximum distortions of lengths in approximately equidistant projections less than 0.7% to that on the maps imperceptible.

References

[1] LAPAINE M, USERY E L. Map projections and reference systems [M]//ORMELING F, RYSTEDT B. The World of Maps. Stockholm: International Cartographic Association, 2014.

[2] LAPAINE M. Map projection article on wikipedia[C]//Proceedings of the 29th International Cartographic Conference (ICC 2019). Tokyo: ICA, 2019.

[3] SNYDER J P. Map projections: a working manual[R]. Washington: U.S. Government Printing Office, 1987.

[4] LAPAINE M, FRANČULA N, TUTEK Ž. Local linear scale factors in map projections in the direction of coordinate axes[J]. Geo-spatial Information Science, 2021, 24(4): 630-637. DOI: 10.1080/10095020.2021.1968321.

[5] LAPAINE M. Local linear scale factors in map projections of an ellipsoid[J]. Geographies, 2021, 1(3): 238-250. DOI: 10.3390/geographies1030014.

[6] EULER L. De repraesentatione superficiei sphaericae super plano [J]. Acta Academiae Scientiarum Imperialis Petropolitanae, 1777(1): 107-132.

[7] BIERNACKI F. Theory of representation of surfaces for surveyors and cartographers [M]. Washington: US Department of Commerce and the National Science Foundation, 1965.

[8] YANG QIHE, SNYDER J P, TOBLER W R. Map projection transformation: principles and applications[M]. London: Taylor & Francis Press, 2000.

[9] LAPAINE M, FRANČULA N. Map projections classification[J]. Geographies, 2022, 2(2): 274-285. DOI: 10.3390/geographies2020019.

[10] GINZBURG G, SALMANOVA T. Atlas dlia vybora kartograficheskikh proektsii. Trudy Tsentral'nogo nauchno-issledovat'skogo instituta geodezii, aeros'emki i kartografii.. Vypusk 110[M]. Izd-vo Geodezicheskoi literatury: Moscow, 1957.

[11] GINZBURG G A. Zritelnaya otsenka kartograficheskikh izobrazheniy [J]. Ucheni zapiski Hark'v'skogo derzhavnogo univesitetu im. O. M. Gorkogo, 1940, 18: 67-81.

[12] BUGAYEVSKIY L M, SNYDER J P. Map projections: a reference manual[M]. London: Taylor & Francis Press, 1995.

[13] LAPAINE M, FRANČULA N, VUČETIĆ N. Površina Hrvatskog mora i otoka[C]//Proceedings of the CAD Forum 1993. Zagreb: CAD Sekcija Saveza Društava Arhitekata Hrvatske, 1993: 47-52.

[14] FRANČULA N, LAPAINE M, VUČETIĆ N. Površina Republike Hrvatske na temelju digitaliziranih granica općina[C]//Proceedings of the 38th Medunarodni Godišnji Skup KoREMA. Zagreb: KoREMA, 1993: 372-375.

[15] LAPAINE M, FRANČULA N, VUČETIĆ N. Procjena točnosti površina odredenih na temelju digitaliziranih granica[C]//Proceedings of the 39th Medunarodni Godišnji Skup KoREMA. Zagreb: Elektrotehnički Fakultet, 1994: 246-249.

[16] DUPLANČIĆ LEDER T, UJEVIĆ T, ČALA M. Coastline lenghts and areas of islands in the Croatian part of the Adriatic Sea determined from the topographic maps at the scale of 1 : 25 000[J]. Geoadria, 2004, 9(1): 5-32.

[17] FRANČULA N. Mjerenje duljina i površina u Google Mapsu i Google Earthu[J]. Kartografija i Geoinformacije, 2020, 19(33): 114-119.

[18] KAVRAYSKIY V V. Izabranie trudy, Tom II: matematicheskaya kartografiya, Vypusk 1[M]. Obshchaja teorija kartograficheskikh proyekciy: Izdanie Upravleniya nachal'nika Gidrograficheskoj sluzhbi VMP, 1958.

[19] LAPAINE M, FRANČULA N. Map projection aspects[J]. International Journal of Cartography, 2016, 2(1): 38-58. DOI: 10.1080/23729333.2016.1184554.

[20] FRANČULA N, JOVIČIĆ D, ŽARINAC-FRANČULA B, et al. Conformal mapping of a rotational ellipsoid onto a sphere and vice versa[C]//Proceedings of the 37th International Annual Gathering KoREMA. Zagreb: KoREMA, 1992: 268-272.

[21] FRANČULA N, JOVIČIĆ D, ŽARINAC-FRANČULA B, et al. Konformno preslikavanje rotacijskoga elipsoida na sferu i obratno primjenom trigonometrijskih redova[J]. Geodetski List, 1992, 46(2): 181-189.

[22] LAPAINE M, ŽARINAC-FRANČULA B, FRANČULA N, et al. Ekvivalentno preslikavanje rotacijskoga elipsoida na sferu i obratno primjenom trigonometrijskih redova[J]. Geodetski List, 1993, 47(4): 325-332.

[23] LAPAINE M, ŽARINAC-FRANČULA B, JOVIČIĆ D, et al. Ekvidistantno preslikavanje po meridijanima rotacijskoga elipsoida na sferu i obratno primjenom trigonometrijskih redova[J]. Geodetski List, 1994, 48(4): 351-359.

[24] LAPAINE M, FRANČULA N. Web-Merctorova projekcija-jedna od cilindričnih projekcija elipsoida u ravninu[J]. Kartografija i Geoinformacije, 2021, 20(35): 30-47.

[25] ZINN N. Web mercator: non-conformal, non-mercator[EB/OL]. [2020-06-26]. http://www.hydrometronics.com/downloads/Web%20Mercator%20-%20Non-Conformal,%20Non-Mercator%20(notes).pdf.

[26] FRANČULA N. Kartografske projekcije[D]. Zagreb: Geodetski Fakultet, 2000.